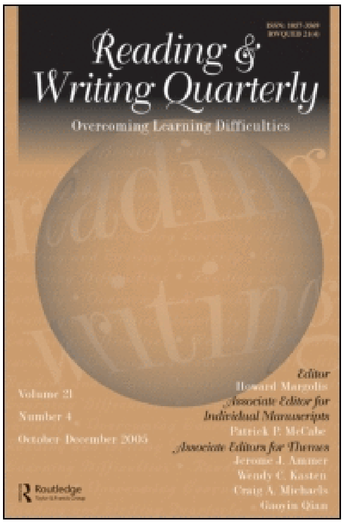

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THE LINGUISTIC CHALLENGES OF MATHEMATICS TEACHING AND LEARNING: A RESEARCH REVIEW

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This article synthesizes research by applied linguists and mathematics educators to highlight the linguistic challenges of mathematics and suggest pedagogical practices to help learners in mathematics classrooms. The linguistic challenges include the multi-semiotic formations of mathematics, its dense noun phrases that participate in relational processes, and the precise meanings of conjunctions and implicit logical relationships that link elements in mathematics discourse. Research on pedagogical practices supports developing mathematics knowledge through attention to the way language is used, suggesting strategies for moving students from informal, everyday ways of talking about mathematics into the registers that construe more technical and precise meanings.

This article synthesizes research by applied linguists and mathematics educators to highlight the linguistic challenges of mathematics learning and suggest some pedagogical practices that may help learners develop mathematical understanding. Since at least the mid-1980s, researchers have been pointing to ways that language is implicated in the learning of mathematics. These studies identify linguistic structures that are used in mathematics in different ways from how language is typically used in everyday life, suggesting that these forms present challenges to many students (e.g., Adams, 2003; Pimm, 1987; Spanos, Rhodes, Dale, & Crandall, 1988). More recently, linguists who take a functional perspective on language have deepened our

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understanding of the linguistic challenges of mathematics education by describing the grammatical patterns through which mathematical knowledge is construed (e.g., Lemke, 2003; O'Halloran, 1999, 2000, 2003; Veel, 1999).

As with all school learning, a key challenge in mathematics teaching is to help students move from everyday, informal ways of construing knowledge into the technical and academic ways that are necessary for disciplinary learning in all subjects. Each subject area has its own ways of using language to construct knowledge, and students need to be able to use language effectively to participate in those ways of knowing. The linguistic challenges of mathematics education were highlighted by M.A.K. Halliday (1978) in his influential discussion of the "mathematical register." He pointed out that counting, measuring, and other "everyday" ways of doing mathematics draw on "everyday" language, but that the kind of mathematics that students need to develop through schooling uses language in new ways to serve new functions. This is not just a question of learning new words, but also new "styles of meaning and modes of argument . . . and of combining existing elements into new combinations" (Halliday, 1978, pp. 195–196). Halliday defined *register* as

> a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

This notion of a mathematical register helps us understand the ways that language constructs mathematical knowledge in different ways than it constructs other academic subjects.

Learning the language of a new discipline is a part of learning the new discipline; in fact, the language and learning cannot be separated. Because students come to school with an everyday language with which they have constructed their knowledge of the world, the school can build on that knowledge and language and move students toward new and more scientific and technical understandings by being aware of the linguistic challenges that accompany the conceptual challenges of learning. This article reports on work that has described some common patterns of mathematics language and on research that recommends ways that teachers can develop students' ability to understand and use these patterns.

FEATURES OF THE MATHEMATICS REGISTER

Researchers in systemic functional linguistics (SFL), Halliday's elaborated grammar of English that links meaning with the grammatical forms through which meaning is made (Halliday & Matthiessen, 2004; Schleppegrell, 2004), have described the linguistic features of the mathematics register that construct this technical discourse. Table 1 lists some of its key features, based on the works of Lemke (1989, 2003), O'Halloran (1999, 2000, 2003), and Veel (1999).

Multiple Semiotic Systems

In doing mathematics, it is not enough to be able to work with the language alone; mathematics draws on multiple semiotic (meaning-creating) systems to construct knowledge: symbols, oral language, written language, and visual representations such as graphs and diagrams. In addition, it uses features such as order, position, relative size, and orientation in meaningful ways (Pimm, 1987). Because concepts that mathematics construct are often difficult to articulate in ordinary language, mathematics symbolism has developed to express meanings that go beyond what ordinary language can express. For example, mathematics symbolism can be used to describe relationships of parts to whole, and to construct trends and patterns of continuous covariation that cannot be presented as precisely in natural language. Visual displays, in the form of graphs and diagrams, can represent the information presented in the mathematics symbolism in ways that language cannot (O'Halloran, 1999).

There are advantages for each type of display of meaning, whether in natural language, mathematics symbolism, or the visual display of

Table 1. Features of the classroom mathematics register

Multiple semiotic systems
- mathematics symbolic notation
- oral language
- written language
- graphs and visual displays

Grammatical patterns
- technical vocabulary
- dense noun phrases
- *being* and *having* verbs
- conjunctions with technical meanings
- implicit logical relationships

diagrams and graphs. O'Halloran (1999) shows how a trigonometry problem uses natural language, mathematics symbolism, and graphic representations, requiring students to be able to recognize the meanings in the interaction of these semiotic systems. The problem uses a diagram representing a man on a cliff looking down at a river and wanting to calculate the width of the river with a rope and device that measures angles (O'Halloran, 1999, p. 24). Language provides the contextual information about the situation, the mathematics symbolism describes the pattern of relationships between the entities, and the diagram provides a connection between the material world (a cliff and a river) and the mathematical processes that are constructed in the problem, a connection that was formulated in oral language in the classroom. Thus, the written language, the mathematics symbolic statements, the visual representation, and the oral language work together to construct meaning as the teacher and students interact in discussing the problem.

Language, mathematical expressions, and visual diagrams, as well as the gestures and actions of participants in the classroom, together construct meaning, and "…it is only by cross-referring and integrating these thematically, by operating with them as if they were all component resources of a single semiotic system, that meanings actually get effectively made and shared in real life" (Lemke, 2003, p. 229). Thus, learning mathematics is not just a question of manipulating symbols, but of understanding how different systems for making meaning interact. Lemke points out that

> too much opportunity for gaining mathematical understanding and intuition, too much practice at learning how to use mathematical meaning in real situations, is lost if mathematics is not taught, particularly at the introductory level, as a co-equal partner with language and visual representation in the analysis of natural and social phenomena. (p. 231)

The Grammar of Mathematics

Mathematics is highly *technical*, with characteristic patterns of vocabulary and grammar. The technical vocabulary includes mathematics words such as *sum* or *fraction*, but also words that are not solely mathematical but have particular meanings in mathematics, such as *place, borrow*, and *product*. Learning the new vocabulary that is centrally mathematical may be easier than learning the technical meanings for words that students already know in other contexts. In addition, students need to be able to use the technical vocabulary in meaningful patterns of language in mathematics. Just knowing mathematical words such as *more, less*, and *as many as*, for example, is not enough; students also need to learn the language patterns associated with these words and how they construct concepts in mathematics.

For example, if students only construct mathematics concepts in everyday language, the relationships that they construe may be technically incorrect. MacGregor (2002, p. 4) provides an example of a student who says, *if Tina has twice as much money as George, then George has twice as less than Tina*. She points out that the student is using a grammatical form common in everyday life, quoting a newspaper that reports *traffic in Sydney during peak hours is nine times slower than in Melbourne*. These comparative statements are composites, "blurring the distinction between the concepts of difference and proportional comparison" (MacGregor, 2002, p. 4). MacGregor found that "students who described a relation between numbers in an informal, unclear or immature way were unable to relate it to a mathematical operation" (2002, p. 1). This makes it crucial for teachers' and students' talk to apprentice students into the technical language of mathematics.

Teachers typically recognize the technical vocabulary as a challenge, but may not be aware of the grammatical patterning that technical vocabulary brings with it. This grammatical patterning includes the use of long, dense noun phrases such as *the volume of a rectangular prism with sides 8, 10, and 12 cm*. Such noun phrases often have pre-numerative phrases that name an abstract, but quantifiable, mathematical attribute of the head noun (e.g., *the volume of, the length of*), classifying adjectives that precede the noun (e.g., *prime* number; *rectangular* prism), and qualifiers that come after the noun (e.g., a number *which can be divided by one and itself*; prism *with sides 8, 10, 12 cm*; examples from Veel, 1999).

These long noun phrases then participate in constructing complex meaning relationships in the problems students have to solve. Typically, they participate in *relational processes* constructed in clauses with *be* and *have* (Veel, 1999). These verbs construct different kinds of relational processes that are common in mathematics, *attributive* and *identifying* processes. An attributive process constructs information about membership in a class or part-whole relationship (e.g., *A square is a quadrilateral* or *Three and four are factors of twelve*). An identifying process, on the other hand, constructs relationships of identity and equality, as in *A prime number is a number that can only be divided by one and itself* or *The mean, or average, score* *is the sum of the scores divided by the number of scores*. What is key is that the attributive clause is non-reversible; it is not true to say that *A quadrilateral is a square*, or *The factors of twelve are three and four*, whereas the identifying clauses construct relationships of equality, and hence, are reversible. So we can say *A number that can only be divided by one and itself is a prime number* or *The sum of the scores divided by the number of scores is the mean, or average, score* (examples from Veel, 1999, p. 195). Attributive clauses classify objects and events, while identifying clauses define technical terms and provide a bridge between technical and less technical ways of presenting knowledge in mathematics by enabling two formulations to be presented as equivalent (for example, *Sides of the triangle that are in the same positions are corresponding sides of the triangles*). Relational processes are also a feature of the multiple choice questions that are often used to assess students' mathematics knowledge on standardized tests, as they ask *which of the following is correct/true/the best way*, etc. (Veel, 1999). The verbs *be* and *have* and other related verbs (*means, equals*, etc.) are challenging in their grammatical features. Students with first languages other than English may be accustomed to constructing relationships of attribution and identity in different ways than English. In Spanish, for example, the verb *is* has two different forms, construing different meaning relationships.

Working out the meanings of these relational constructions can be difficult, as Moschkovich (1999) shows in her description of a third grade classroom with English language learners who are discussing the geometric shapes they are forming from a tangram puzzle. The students are grappling with both notions of class inclusion (Are these all parallelograms?) and of properties (Do they have parallel sides?) as they sort out the relationships among a square, a rectangle, a parallelogram, and a trapezoid. Using language like *It has parallel sides* and *It is a parallelogram* to construct the notions of property vs. category, at one point the teacher interprets a child's *Does not have parallelogram* as *Is not a parallelogram* (Moschkovich, 1999, p. 15). Moschkovich points out that the student and teacher are talking about these tangrams from different points of view; the teacher is referring to categories while the student is talking about the properties of one of the figures. This difference in point of view is not always apparent in the classroom conversation.

Another challenge of the mathematics register is the precise and technical meanings of conjunctions that may be used in different ways in ordinary everyday language. In word problems, and in developing theorems and proofs, conjunctions such as *if, when*, and *therefore* are used in precise ways, and constructions such as *given* and *assume* take on new roles. Conjunctions often link clauses in complex ways, with variations presenting similar meanings.

In addition, the mathematical operations that are used to construct mathematical reasoning are sometimes left implicit (O'Halloran, 1999, 2000). O'Halloran shows how solving mathematics problems often involves "long chains of reasoning that provide little or no indication of the results, definitions, axioms, operational properties or laws that have been used" (2000, p. 377). For example, in geometry proofs, the various properties and postulates that underlie the argument made in the proof are not spelled out, but rather are assumed to have been already learned and internalized. Chapman's (1995) linguistic analysis illustrates that even in mathematics textbooks, these underlying principles are often left implicit.

Summary and Implications

The linguistic aspects of mathematics that distance it from ordinary use of language include the multiple semiotic systems that bring together symbolic representations and visual images that do not match up exactly with their "translation" into the oral and written language used to develop the meanings they present. In addition, the technical vocabulary and grammatical structuring associated with it make the oral and written language challenging in its own right. The grammatical patterning brings together long, dense noun phrases in clauses and sentences constructed with *being* and *having* verbs that present a variety of meaning relationships. In addition, mathematics problems often use conjunctions that have meanings different from their everyday uses, or include implicit logical relationships that are not spelled out.

Although the features of the mathematics register in Table 1 have been presented as separate elements, in the learning and teaching of mathematics, of course, they are always used in interaction with each other. O'Halloran (2003, p. 196) illustrates this. She shows how a mathematics problem that is presented in math symbolism requires dense nominal structure when translated into words. Here is the problem:

$$a^2 + (a + 2)^2 = 340$$

In written language, this equation can be represented as:

The sum of the squares of two consecutive positive even integers is 340.

As O'Halloran points out, what the language encodes as one *thing*, in the dense noun phrase *The sum of the squares of two consecutive positive even integers*, is represented in mathematical symbolism as a series of *processes;* squaring a, squaring $a+2$, and adding those products together, and that this *thing* is then equated, using *is*, with 340. As we have seen, the grammatical patterning of mathematics often presents *processes* as if they were *things* by construing them as nouns and noun phrases. The distinctive mathematics operations, for example, such as *addition, subtraction*, and *multiplication*, are *processes*, but the grammar constructs them as *things* (in noun phrases). This makes mathematics a very *objectified* discourse (Sfard & Lavie, 2005), and students need to be able to recognize the relationship between the *things* of the grammar and the *processes* of the mathematics reasoning. In the symbolic solution, students have to recognize these distinct processes and how their elements are split apart and then recombined and reconfigured. The brackets, orderings, and spatial display are all part of how the mathematical meaning is constructed.

In the classroom, the teacher uses oral language to discuss the equation and its solution, adding another layer of linguistic complexity, as the oral language does not exactly capture the relationships in the ways the written language or symbolic language does. O'Halloran (2000, p. 384) reports that in this case, the teacher said:

> ...**and then you've got to add on the 'a' squareds because of the brackets and the squareds, add up the 'a' squareds so you get two 'a' squareds plus your four 'a'**

The teacher's oral language again presents the elements in the mathematical symbol statements as *things* to be manipulated, even though the notion of *square root*, for example, is not a *thing*, but a process or operation. Translating among all these semiotic resources and maintaining the technical register is a challenge for teachers and students.

This summary of the linguistic challenges of the mathematics register, based on functional linguistics research, highlights the need to expand our understanding of the language issues in mathematics classrooms beyond a focus on vocabulary or specialized terminology. Mathematics reasoning uses patterns of language that draw on grammatical constructions that create dense clauses linked with each other in conventionalized ways that are different from ordinary informal language use. At the same time, the mathematics register draws on a range of modalities, constructing meaning by deploying multi-semiotic resources that interact with each other. These language challenges need to be taken into account in the mathematics classroom. The next section of this article synthesizes research related to language in the mathematics classroom and shows how it points to ways that teachers can support students' development of the mathematics register.

SUPPORTING THE DEVELOPMENT OF THE MATHEMATICS REGISTER AND MATHEMATICAL KNOWLEDGE

The linguistic challenges of mathematics suggest that focusing on the features of the language through which mathematics is constructed can be a strategy for engaging students and supporting their learning. By helping students recognize how mathematics is constructed in multi-semiotic and grammatical resources, teachers can support students' development of the technical construal of knowledge that has to be achieved for mathematics learning. The technical construal is important for them to perform well on assessments, but the development of the mathematics register has to build from oral language that moves from the everyday to the technical. As with all language development, students need opportunities to use the mathematics register in interactive activities in which they construct meaningful discourse about mathematics. They need to practice the multi-semiotic construction of meaning, drawing on all modalities. This section discusses research that suggests that teachers play an important role in helping students use language effectively in learning and demonstrating their mathematical knowledge.

Perhaps more than in any other discipline, the construction of knowledge about mathematics depends on the oral language explanations and interaction of the teacher. Veel (1999) reports that teacher spoken language predominates in mathematics classes, and the teacher's words are needed to interpret the meanings that the visual and symbolic representations construct, as "it is spoken language which provides the link between the symbolic and visual representations for students, and is therefore a powerful agent in the learning process" (p. 189). The explanations textbooks provide tend to be dense, so the teacher plays a key role in helping students learn to negotiate the symbols, diagrams, and technical language. On experimental tasks, Leung, Low, and Sweller (1997) found that students benefited from verbal explanations of mathematics problems, at least until they gained a greater facility with solving these problems. As students got more practice, the verbal explanation became less important, indicating the scaffolding role that talk about mathematics may play.

Students develop mathematics concepts as they use them discursively to construe meaning, but as Sfard and Lavie (2005) point out, "[t]o become aware of this discourse's advantages one has to use it; yet, to have an incentive to use it, one has to be aware of the prospective gains of this use" (p. 288). Working with experienced interlocutors is the only way to accomplish this. O'Halloran (2000) recommends that teachers use oral language to unpack and explain the meanings in mathematics symbolism as a way of using the multisemiotic nature of mathematics to help students draw on the different meaning-making modes for understanding. Explicitly focusing students' attention on the linguistic features can help students explore and clarify the technical meanings. This does not mean just talk for talk's sake; teachers need to give attention to when the technical talk can help students develop the mathematics register (Sfard et al., 1998).

As Moschkovich (1999, p. 11) points out, students need to participate both orally and in writing by "explaining solution processes, describing conjectures, proving conclusions and presenting arguments." One way to encourage students' development of extended ways of talking about math is by having students talk with each other. As putting students into groups to discuss mathematics concepts is also a way of limiting teacher talk, group work in math classrooms has become quite common. But while interaction with peers can achieve some goals of the mathematics teacher, interaction with peers alone will not lead to the development of the mathematics register. Students working in groups are not always able to express their ideas clearly or understand each other's explanations (MacGregor, 2002), and several studies of the language students use in such interaction demonstrate that even when they arrive at the right answers, students may not be building the internal understanding that comes through using appropriate technical language to construct the mathematical meanings (Chapman, 1995; MacGregor, 2002; Veel, 1999).

Students do not take up the technical language of mathematics merely by being exposed to it through the teacher's talk or textbook. Veel (1999) compares the student use of mathematical language and teacher/textbook use of mathematical language and finds a major gap between them in features that distinguish mathematics language from everyday language. The teacher's language has higher lexical density, a greater percentage of relational processes, and more long noun phrases, the language resources that construct the mathematics register and make meanings in the technical language needed to understand the concepts. Huang, Normandia, & Greer (2005) analyze the knowledge structures (Mohan, 1986) that appear in teacher and in student discourse in a mathematics class where the teacher expected technical language from the students and encouraged them to talk about math. Students, working in groups, "...could easily describe an equation or a graph, sequentially tell about procedures they have followed to solve a function, and suggest a method or solution. However, whenever students were pushed to reference relevant concepts or principles, explain a method used, or justify a decision made for either a method or solution, they frequently seemed to hesitate or to appear less capable" (Huang et al., 2005, p. 44). Huang et al. found that knowledge structures such as *classification, principles*, and *evaluation* were only used by the teacher. Students used only *description, sequence*, and *choice* knowledge structures, even when pushed by the teacher. The authors suggest that students need explicit instruction in articulating principles to move them beyond the practical aspects of math knowledge in their discussion. They recommend that students be asked to "talk their way into habits of expressing higher-level knowledge structures" (Huang et al., 2005, pp. 44–45), and that teachers integrate thinking and talking at all levels.

Adams (2003) suggests that teachers can move students from the everyday language into the mathematics register by helping students recognize and use technical language rather than informal language when they are defining and explaining concepts; by working to develop connections between the everyday meanings of words and their mathematical meanings, especially for ambiguous terms, homonyms, and similar-sounding words; and by explicitly evaluating students' ability to use technical language appropriately. One way to evaluate this ability is by having students talk about mathematics as they solve problems, encouraging them to articulate patterns and generalizations. This is a context for developing the mathematics register by asking students to talk formally, such as by identifying the referents of *it* and *this one* and referring to numbers or symbols instead of the pronouns and demonstratives that are typical of contexts where everyone can see the symbols on the blackboard (Pimm, 1987).

Pimm (1987) suggests that being able to articulate the patterns formally is key for learning algebra, and that having students verbalize their thinking gives teachers an opportunity to see what students understand and can do, as well as enabling students to get feedback on whether or not they have understood. Students need to recognize, for example, that algebra is not learned by just substituting letters for numbers in solving equations, but instead needs to be understood as a means of dealing with *properties* of numbers and quantities (Pimm, 1987). The meanings that the forms represent are the key to understanding. Pimm also notes that "eliciting mathematical talk focuses attention on argument and conviction by means of explanation, as well as on the task of finding more precise and succinct expressions which may therefore be more readily worked with and verified" (1987, p. 48). As Pimm points out, although people talk about how precise mathematics language is, the precision actually is not in the language itself, but in the way it is used. The technical language has to be practiced and developed along with the mathematics concepts.

This is illustrated in Khisty & Viego's (1999) report on a fifth grade teacher of Latino children who was successful in helping her students develop math understanding. The children read a problem on the blackboard and discussed its meaning before starting to work. Then they worked individually or in pairs to solve the problem, guided by the teacher's questions. When all students had solved the problem, volunteers were asked to present their solution to the class. The discussion focused not on the answer, but on the reasoning the students used. The teacher used technical language, creating "experiences in which these words are simply used to express meaning in mathematics" (p. 78). In this case, the students took up these ways of using language.

"Revoicing" is a technique for interaction discussed by Moschkovich (1999). She provides examples of this strategy:

Student: **"The rectangle has par . . . parallelogram . . . and the triangle does not have parallelogram."**
Teacher: **"He says that this [a triangle] is not a parallelogram"**

Moschkovich suggests that such revoicing supports student participation by indicating the acceptance of the student's contribution and by keeping the discussion mathematical, as the student's contribution is reformulated in ways that are "closer to the standard discourse practices of the discipline" (1999, p. 15). She notes that it is not unproblematic to revoice students' contributions, however, as the teacher's interpretation of what the student means may not always be correct. However, this strategy does enable student participation because it focuses on the student's contribution to mathematical discussion and indicates a stance that is focusing on meaning rather than form. This focus on meaning, not form, is key to all discussion about mathematics concepts.

Different opportunities present themselves in mathematics classrooms for talk of different kinds. Setati (2005) describes four kinds of talk that she observed in a grade four multilingual classroom in South Africa, following Moschkovich (2002) and Gee (1999). Two of these are explicitly constructing mathematical knowledge: *procedural* talk that lays out the steps taken to solve a problem, and *conceptual* talk about the reasons for calculating in particular ways or for using particular procedures. In addition, teachers use *regulatory* talk for classroom management and *contextual* talk to bring in background information when students are solving word problems. An awareness of these different contexts of language use can enable teachers to think about the different opportunities they offer for responding to and validating the everyday language the students bring to the classroom, while seizing opportunities to move the students in the direction of the more technical mathematics register when constructing mathematical knowledge. For example, in contextual and regulative contexts, teachers may foreground the more everyday language, while during procedural and conceptual discussion, the more technical language is highlighted.

Moving between the everyday and the technical is not clear-cut, however, and raises what Adler (1997) calls the *dilemma of mediation*. As she points out, teachers need to both listen to and validate the perspectives learners bring, while at the same time moving them from informal to formalized discourse. In her case study of a South African classroom where the teacher uses both student grouping and teacher-student interaction, Adler shows how the *withdrawal* of the teacher enables a participatory classroom culture, but "her *mediation* is essential to improving the substance of communication about mathematics and the development of scientific concepts" (p. 255). Adler suggests that the dilemma is "in shaping informal, expressive and sometimes incomplete and confusing language, while aiming toward the abstract and formal language of mathematics," pointing out that "a participatory-inquiry approach, and the possibilities it offers for learner activity and pupil-pupil interaction, can inadvertently constrain mediation of mathematical activity and access to mathematical concepts" (p. 236).

The notion of explicit teaching, and making the language transparent in its meaning, is also not a straightforward issue. Adler (1998) demonstrates that explicit mathematics language teaching, focused on instructions and explanations, helped all learners, but the teachers she interviewed felt uncomfortable with all the talking they were doing. This is an issue Adler (1998) calls the *dilemma of transparency*; whether to talk or not to talk, and like the *dilemma* *of mediation*, this is not an issue that can be resolved; instead, it is one of the tensions that teachers face.

While oral language is dominant in mathematics classrooms, reading and writing also play roles in teaching and learning. Students can read examples or definitions, read through and talk about assignments and expectations, and read and work through texts, including graphic and visual texts, to share questions and responses and make sense of the language (Siegel & Fonzi, 1995). In reading word problems out loud together, for example, teachers and students make the text a part of classroom dialogue, elaborating and commenting on what it says so that students become more comfortable talking about the meanings (Adams, 2003; Chapman, 1995; Lemke, 1989). Explicitly attending to features of the language itself may also be necessary (Schleppegrell, 2004). Attention can be paid to unpacking the meanings in the dense noun phrases and clarifying relationships that are constructed in the verbs and conjunctions, as well as by making explicit what might have been left implicit in the formulation of the problem. Staub and Reusser (1995) also suggest a clearer consideration of the situations in the problems and their relevance to instructional goals. As Veel (1999) points out, most word problems are contrived by teachers or textbook writers to fit the particular calculation skills that are in focus. Although they purport to make mathematics real, or to connect with students' actual lives, in fact, they often do not address the everyday experience of most students.

Writing tasks also have a place in mathematics instruction, but as Marks and Mousley (1990) point out, it is important for teachers to ask students to write in factual rather than narrative modes. Instead of writing "math stories," for example, students can write authentic genres of mathematics such as the presentation of procedures, descriptions and classes of things, explanations of judgments or findings, and arguments about theorems and other mathematical tasks (see also Solomon & O'Neill, 1998). These recommendations are supported by research by Johnson et al. (1998) in a fifth grade mathematics class. Students wrote journals that retold what they had done, and the researchers found that most students gained more in writing skills than in the mathematical thinking, which was the teacher's goal in having students write. Fortescue (1994) reports on a third grade teacher who asked children to describe a mathematics activity without providing instruction about *how to write*. A selection of the children's responses were vague or not descriptive, often including only feelings or the outcome of the activity, even though the teacher had wanted them to focus on the process and elements involved in solving a problem. After the teacher began to model talking and writing about the activity jointly with the students and had students read their classmates' procedures and try to do the activity based on their texts, the children all showed improvement over time, with 70% of the children saying it helped them learn mathematics. Fortescue notes that "students needed many experiences communicating about math in order to write about it effectively" (p. 580), and that whole group modeling and peer interaction with mathematics before writing is what made the writing effective. In addition, the teacher's much clearer goal for the writing also made it a more effective activity. The writing must be technical mathematics language, to the extent the students are capable, if it is going to foster the development of mathematics knowledge.

Learning mathematics and the language of mathematics is a challenge for all students, but is especially challenging for students who have no opportunities to use academic language outside of school. Such students include speakers of nonstandard varieties of English and students for whom English is a second (or other) language. For these students, initiation into the ways of using language that are valued is key to enabling their participation in increasingly complex and abstract learning contexts (Colombi & Schleppegrell, 2002). Teachers can facilitate this if they use the mathematics register effectively and work to build language across a unit of study in deliberate ways, moving from the everyday to the technical construal of mathematics knowledge, using spoken language, reading, and writing. Lemke (2003) suggests that teachers should translate back and forth between the ordinary and technical language, embed the uses of mathematics in application contexts, and expose students to real out-of-school settings for use of mathematics (see also Sáenz-Ludlow & Walgamuth, 2001).

Although it may seem obvious that teachers need to know the mathematics register themselves and use it, this issue is not completely straightforward, especially in multilingual contexts. If the language of instruction is not the students' home language, teachers face additional demands in supporting the development of the school language and the mathematics register. In some cases, the students' home language may not have a well-developed mathematics register, making it impossible for the teacher to use the home language for mathematics instruction. Any language has the potential to develop registers for teaching mathematics, but for historical reasons, not all languages have been developed for this purpose. Adler (1998), for example, reports on a South African classroom where the teacher "runs out of words" when trying to explain advanced math in Tswana. Language policy can be oriented to provide resources that would enable the mathematics register to be developed in any language through which children are instructed, but until this is done, teachers may face difficulties in attempting to translate math concepts into students' home languages.

When the students' first language does have a fully developed mathematics register, as in the case of Spanish, the teacher's expertise with the mathematics register *in the language of instruction* is an important consideration. A student (or teacher) might know a language without being able to use it to express mathematics knowledge. If teachers have not learned the mathematics register in the language of instruction, they may not be in a position to scaffold students' development of that register and the accompanying mathematical concepts. To use the mathematics register at the appropriate level of technicality, a bilingual educator needs to be bilingual in mathematics. Knowing a language means more than knowing technical terms, and having a bilingual translation is not sufficient for scaffolding the development of mathematics language in a second language (Gutierrez, 2002). A Spanish bilingual teacher, for example, would need to have studied mathematics in Spanish to be able to teach it in Spanish. Gutierrez reports that many bilingual teachers are unprepared to teach mathematics in Spanish.

Learning the mathematics register takes time, and teachers need to set goals that scaffold the development of precise ways of using language over lessons and units of study. Chapman (1995) shows how this can be done, tracking the understanding of a concept over several lessons and showing how it is built up in teacher talk, whole-class teacher-student interaction, interaction between students, and reading of the textbook. The teacher focuses students on using appropriate language to construct the concept and contextualizes the textbook language through spoken language to help students understand the relationships in the textbook definitions. Chapman also shows that when the students work out a problem together in groups, they construct the concepts in ways that are different from the teacher or the textbook, and they sometimes talk past each other. This underscores the need for teachers to continually make explicit reference to the technical language through which the mathematics knowledge is built up, helping the students learn by making connections with other texts and other contexts. We need more research on language in mathematics classrooms, as well as research that describes the learning process over a unit of instruction, so the movement from everyday to more technical language can be tracked.

CONCLUSIONS

The notion of a mathematics register and recognition of the role of language in mathematics learning and teaching cast doubt on the common wisdom that suggests that mathematics is the least language-dependent subject. Abedi and Lord (2001), for example, found that English language learners score lower on standardized tests of mathematics than students who are fluent in English, suggesting the language dependence of mathematics. In a report on the teaching of algebra to English learners in California, Lager (2004) notes that fewer than 23% of current English learners and fewer than 50% of English learners who had been re-designated as "fluent" passed the 2002–2003 high school exit exam, on which 36% of the mathematics questions are from algebra. He finds that "the state's current models scarcely address the range of linguistic issues hindering English learners' performance" in algebra (p. 1), and suggests that the more advanced mathematics becomes, the more language-dependent it is.

Even proficient speakers of English face challenges from the language of mathematics. Abedi & Lord (2001, p. 219) also found that "nationally, children perform 10% to 30% worse on arithmetic word problems than on comparable problems presented in numeric format," pointing clearly to factors other than mathematical skill. Language is implicated in teaching and learning mathematics much more than is generally realized. Classroom interaction and activities that use language to construct meaning can play a major role in giving students access to mathematics knowledge, making it accessible to those students who have little access to the implicit learning that takes place outside the classroom for other students (Veel, 1999).

Recognizing the challenges of the mathematics register, Abedi & Lord (2001) have conducted research that analyzed the effect of modifying the language features of mathematics assessment tasks to try to make the meanings more accessible to struggling learners. They changed the released math items from the National Assessment of Educational Progress (NAEP) 1992 in several ways, shortening nominal expressions, making conditional relationships more explicit, changing complex question phrases to simple question words and passive voice to active, and replacing less familiar or frequent non-mathematics vocabulary with more common terms. They then interviewed eighth grade students who worked both the original and modified forms of these problems, asking them whether anything was confusing or easy about the problems and whether they would choose first to do the original or revised problem. Most students chose the revised versions and performed better with those versions.

Low-performing mathematics students benefited more from the revisions than those in higher mathematics and algebra, English Language Learners benefited more than proficient speakers of English, and students identified as having low socioeconomic status benefited more than others. This is promising work that can contribute to the development of materials that provide better support for students who are moving from the everyday language into the more technical. Other research has also demonstrated that the wording of math problems has a major influence on comprehension and children's ability to solve them (Staub & Reusser, 1995).

But it is also important to recognize that students need to learn to deal with the dense and technical language. If mathematics concepts are not introduced and explained in oral language that moves from the ordinary language that students already understand to the more technical language that they need to develop for full understanding of the concepts, student learning suffers. Raiker (2002), for example, found that the second through fourth grade students in her study had difficulty learning if the technical meaning of mathematical words such as *division* was not established. This was especially significant because the study also found that teachers did not typically focus on language or plan for teaching the language required for explanations of mathematical concepts. O'Halloran (2003) found that teachers of working class and female students used more informal and non-technical language in the oral discourse of the classroom, and suggests that there are social class and gender implications for access to knowledge when teachers use less technical language.

Of course, the challenges of mathematics go beyond the language issues, but the linguistic challenges need to be addressed for students to be able to construct knowledge about mathematics in the ways that can ensure their success. We have seen that features of the mathematics register can be identified and analyzed by students to see how meaning is made in mathematics. Teachers can support the development of the multi-semiotic mathematics register through oral language that moves from the everyday to the technical mode. Students can be encourage to produce extended discourse in mathematics classrooms, engage in discussion about the language through which word problems are constructed, and practice the writing of mathematics concepts in authentic ways. Teachers can become aware of the linguistic issues in learning and teaching mathematics and can develop tools for talking about language in ways that enable them to engage productively with students in constructing mathematics knowledge. Further research by applied linguists and mathematics educators can explore the linguistic challenges of mathematics learning in its multi-semiotic complexity to provide more support for teachers who want to engage struggling learners.

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